Newton's forward interpolation formula is
\[ y = y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0 + \ldots \]
Differentiating both sides w.r. to \( p \) we have,
\[ \frac{dy}{dp} = \Delta y_0 + 2p-1 \Delta^2 y_0 + \frac{3p^2-6p+2}{2!} \Delta^3 y_0 + \ldots \]
But \( p = x - x_0 \), \( \frac{dp}{dx} = \frac{1}{h} \)
Now
\[ \frac{dy}{dx} = \frac{dy}{dp} \cdot \frac{dp}{dx} = \frac{1}{h} \left[ \Delta y_0 + 2p-1 \Delta^2 y_0 + \frac{3p^2-6p+2}{2!} \Delta^3 y_0 + \ldots \right] \]......(1)
At \( x = x_0 \), \( p = 0 \). Hence putting \( p = 0 \),
\[ \frac{dy}{dx} \text{ at } x_0 = \frac{1}{h} \left[ \Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \ldots \right] \]
Again differentiating (1) w.r.to \( x \) we get,
\[ \frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{1}{h} \left[ 2/2! \Delta^2 y_0 + 6p-3 \Delta^3 y_0 + \frac{12p^2-36p+22}{3!} \Delta^4 y_0 + \ldots \right] \]
Putting \( p = 0 \) we obtain
\[ \frac{d^2y}{dx^2} \text{ at } x_0 = \frac{1}{h^2} \left[ \Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12} \Delta^4 y_0 - \frac{5}{6} \Delta^5 y_0 + \frac{137}{180} \Delta^6 y_0 + \ldots \right] \]......(1)
\[ \frac{d^3y}{dx^3} = \frac{1}{h^3} \left[ \Delta^3 y_0 - 2/3 \Delta^4 y_0 + \ldots \right] \]......(4)

ii) Newton's backward interpolation formula is
\[ y = y_n + p \nabla y_n + \frac{p(p+1)}{2!} \nabla^2 y_n + \frac{p(p+1)(p+2)}{3!} \nabla^3 y_n + \ldots \]
Differentiating both sides w.r. to \( p \) we have,
\[ \frac{dy}{dp} = \nabla y_n + 2p+1 \nabla^2 y_n + \frac{3p^2+6p+2}{3!} \nabla^3 y_n + \ldots \]
Since \( p = x - x_n \), \( \frac{dp}{dx} = \frac{1}{h} \)
Now,
\[ \frac{dy}{dx} = \frac{dy}{dp} \cdot \frac{dp}{dx} = \frac{1}{h} \left[ \nabla y_n + 2p+1 \nabla^2 y_n + \frac{3p^2+6p+2}{3!} \nabla^3 y_n + \ldots \right] \]......(5)
At \( x = x_n \), \( p = 0 \). Hence putting \( p = 0 \) we get,
\[ \frac{dy}{dx} \text{ at } x_n = 1 / h \left[ \nabla^2 y_n + 2 \nabla^3 y_n + 3 \nabla^4 y_n \ldots \right] \ldots (6) \]

Again differentiating (5) w.r. to x we get,

\[ \frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dp} \right) \frac{dp}{dx} \]

\[ = \frac{1}{h} \left[ \nabla^2 y_n + \frac{6p + 6}{3!} \nabla^3 y_n + \frac{6p^2 + 18p + 11}{12} \nabla^4 y_n + \ldots \right] \]

Putting \( p = 0 \) we obtain

\[ \frac{d^2y}{dx^2} \text{ at } x_n = \frac{1}{h^2} \left[ \nabla^2 y_n + \frac{11}{12} \nabla^3 y_n + \frac{5}{6} \nabla^4 y_n + \ldots \right] \ldots (7) \]

\[ \frac{d^3y}{dx^3} \text{ at } x_n = \frac{1}{h^3} \left[ \nabla^3 y_n + \frac{3}{2} \nabla^4 y_n + \ldots \right] \ldots (8) \]

Problems

1. Given that \( x : 1 \ 1.1 \ 1.2 \ 1.3 \ 1.4 \ 1.5 \ 1.6 \)
\( y : 7.989 \ 8.403 \ 8.781 \ 9.129 \ 9.451 \ 9.750 \ 10.031 \)

Find \( \frac{dy}{dx} \) & \( \frac{d^2y}{dx^2} \) at \( x = 1.1 \) & \( 1.6 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( \Delta y )</th>
<th>( \Delta^2 y )</th>
<th>( \Delta^3 y )</th>
<th>( \Delta^4 y )</th>
<th>( \Delta^5 y )</th>
<th>( \Delta^6 y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>7.989</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>8.403</td>
<td>-0.036</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td>8.781</td>
<td>-0.030</td>
<td>-0.006</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.3</td>
<td>9.129</td>
<td>-0.026</td>
<td>0.000</td>
<td>-0.003</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4</td>
<td>9.451</td>
<td>-0.023</td>
<td>-0.001</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>9.750</td>
<td>-0.018</td>
<td>0.005</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.6</td>
<td>10.031</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We have \( \frac{dy}{dx} \) at \( x_0 = 1 / h \left[ \Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \ldots \right] \]

\[ \frac{dy}{dx} \text{ at } 1.1 = 3.946 \]

\[ \frac{d^2y}{dx^2} \text{ at } x_0 = 1 / h^2 \left[ \Delta^2 y_0 - \frac{11}{12} \Delta^3 y_0 + \frac{5}{6} \Delta^4 y_0 - \frac{137}{180} \Delta^5 y_0 + \ldots \right] \]

\[ \frac{d^2y}{dx^2} \text{ at } 1.1 = -3.545 \]
we use the above difference table and the backward difference operator?
\[
\frac{dy}{dx} \text{ at } x = x_n = \frac{1}{h} \left[ \nabla y_n + \frac{1}{2} \nabla^2 y_n + \frac{1}{3} \nabla^3 y_n + \ldots \right]
\]
\[
\frac{dy}{dx} \text{ at } x = 1.6 = 2.727
\]
\[
\frac{d^2y}{dx^2} \text{ at } x = 1.6 = -1.703
\]

1) Find the first and second derivatives of the function tabulated below at the points x = 2 and x = 1.9

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>0.128</td>
<td>0.544</td>
<td>1.296</td>
<td>2.432</td>
<td>4</td>
</tr>
</tbody>
</table>

2) Find the first and second derivatives of the function tabulated below at the points x = 1.1 and x = 1.0

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>0.128</td>
<td>0.544</td>
<td>1.296</td>
<td>2.432</td>
<td>4</td>
</tr>
</tbody>
</table>

3) Find the first two derivatives of \((x)^{1/3}\) at x = 50 and x = 56 given the table below

<table>
<thead>
<tr>
<th>x</th>
<th>50</th>
<th>51</th>
<th>52</th>
<th>53</th>
<th>54</th>
<th>55</th>
<th>56</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3.684</td>
<td>3.7084</td>
<td>3.7325</td>
<td>3.7563</td>
<td>3.7798</td>
<td>3.8030</td>
<td>3.8259</td>
</tr>
</tbody>
</table>

**DERIVATIVES USING STIRLING'S FORMULA**

\[
\frac{dy}{dx} \text{ at } x = x_0 = \frac{1}{h} \left\{ \frac{1}{2} (\Delta y_0 + \Delta y_1) - \frac{1}{12} (\Delta^3 y_1 - \Delta^3 y_2 + \ldots) \right\}
\]
\[
\frac{d^2y}{dx^2} \text{ at } x = x_0 = \frac{1}{h^2} \left\{ \frac{1}{2} (\Delta^2 y_1 - \Delta^2 y_2 + \ldots) \right\}
\]
\[
\frac{d^3y}{dx^3} \text{ at } x = x_0 = \frac{1}{h^3} \left\{ \frac{1}{2} (\Delta^3 y_1 - \Delta^3 y_2 + \ldots) \right\}
\]

**Problems**

1. Find the first and second derivatives of the function tabulated below at x = 0.6

<table>
<thead>
<tr>
<th>x</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1.5836</td>
<td>1.7974</td>
<td>2.0442</td>
<td>2.3275</td>
<td>2.6511</td>
</tr>
</tbody>
</table>

Since x = 0.6 is in the middle of the table we will use stirling's formula
\[ \begin{array}{ccccccc}
  x & y & \Delta y & \Delta^2 y & \Delta^3 y & \Delta^4 y \\
 0.4 & 1.5836 & 0.2138 & & & \\
 0.5 & 1.7974 & 0.0330 & 0.0035 & & \\
 0.6 & 2.0442 & 0.0365 & 0.0003 & 0.0038 & \\
 0.7 & 2.3275 & & & & \\
 0.8 & 2.6511 & & & & \\
\end{array} \]

By Stirling's formula
\[
\frac{dy}{dx} \text{ at } x = x_0 = \frac{1}{2} \left\{ \frac{1}{h^2} (\Delta y_0 + \Delta y_{-1}) - \frac{1}{12} (\Delta^3 y_{-1} + \Delta^3 y_{-2}) + \frac{1}{60} (\Delta^5 y_{-2} + \Delta^5 y_{-3}) \ldots \right\}
\]
\[
\frac{d^2 y}{dx^2} \text{ at } x = x_0 = \frac{1}{2h^2} \left\{ \Delta^2 y_{-1} - \frac{1}{12} \Delta^4 y_{-2} + \ldots \right\}
\]

\[
\frac{d^2 y}{dx^2} \text{ at } x = 0.6 = 3.6475
\]

Home work

1) Find the value of \( f'(0.5) \) using Stirling's formula from the following data

\begin{align*}
  x & : 0.35 \ 0.4 \ 0.45 \ 0.5 \ 0.55 \ 0.6 \ 0.65 \\
  y & : 1.521 \ 1.506 \ 1.488 \ 1.467 \ 1.444 \ 1.418 \ 1.389
\end{align*}

**NUMERICAL INTEGRATION**

1) **NEWTON – COTES QUADRATURE FORMULA**

\[
I = nh \left[ \frac{y_0 + n \Delta y_0 + n(n-3) \Delta^2 y_0}{2} + \ldots \right]
\]

2) **TRAPEZOIDAL RULE**

\[
\int_{x_0}^{x_0+ph} f(x) \, dx = h \left[ \frac{(y_0+y_n)}{2} + 2(y_1+y_2+\ldots+y_{n-1}) \right]
\]
3) SIMPSON'S ONE-THIRD RULE
\[ \int_{x_0}^{x_n} f(x) \, dx = \frac{h}{3} \left[ (y_0+y_n) + 4(y_1+y_3+......y_{n-1}) + 2(y_2+y_4+......+y_{n-2}) \right] \]

4) SIMPSON'S THREE-EIGHTH RULE
\[ \int_{x_0}^{x_n} f(x) \, dx = \frac{3h}{8} \left[ (y_0+y_n) + 3(y_1+y_2+y_3+......) + 2(y_3+y_6+......) \right] \]

Evaluate \( \int_0^1 \frac{1}{1+x^2} \, dx \) by using

1) Trapezoidal rule
2) Simpson's 1/3 rule
3) Simpson's 3/8 rule

Divide the integral \((0,6)\) into six parts each of width \(h = 1\)

\[ x : \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \]
\[ y : \quad 1 \quad 0.5 \quad 0.2 \quad 0.1 \quad 0.0588 \quad 0.0385 \quad 0.072 \]

By Trapezoidal rule
\[ \int_{x_0}^{x_n} \frac{1}{1+x^2} \, dx = \frac{h}{2} \left[ (y_0+y_6) + 2(y_1+y_2+y_4+y_5) \right] = 1.4108 \]

By Simpson's 1/3 rule
\[ \int_{x_0}^{x_n} \frac{1}{1+x^2} \, dx = \frac{3h}{2} \left[ (y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4) \right] = 1.3662 \]

By Simpson's 3/8 rule
\[ \int_{x_0}^{x_n} \frac{1}{1+x^2} \, dx = \frac{3h}{8} \left[ (y_0+y_6) + 3(y_1+y_2+y_3+y_4+y_5) + 2y_3 \right] = 1.3571 \]

Home work

1) Use trapezoidal rule to evaluate \( \int_0^3 x^3 \, dx \) considering 5 sub-intervals
2) Evaluate \( \int_{-2}^{3} x^2 \, dx \) by trapezoidal rule
3) Evaluate \( \int_{-3}^{3} x^4 \, dx \) by using trapezoidal rule and Simpson's rule
Difference Equations

An equation which expresses a relation between the independent variable, the dependent variable and the successive differences of the dependent variable is called a difference equation.

Eg: \( \Delta^4 y_x + 5 \Delta^4 y_x + 6 \Delta y_x + 18 \ y_x = x + \sin x \) is the difference equation. Since \( \Delta \) is the forward difference and \( y_x \) is a function of \( x \).

Order and degree of the difference equation

The order of a difference equation written in the form free from \( \Delta \)'s, is the difference between the highest and lowest subscripts of \( y \) or arguments of \( y \). Thus the order of

\[
y_{x+3} - 5 \ y_{x+2} + 7 \ y_{x+1} + \ y_x = 10x - x = 3
\]

The degree of a difference equation written in the form free from \( \Delta \)'s, is the highest power of the \( y \)'s.

Eg: \( (E^2 - 5E + 16) y_x = e^x \) is of degree 1

Solution of a difference equation

A solution of a difference equation is a function of its variable which satisfies the difference equation.

General solution of the difference equation

The general solution or complete solution of a difference equation is the sum of two functions.

Linear difference equation

A Linear difference equation with constant coefficients is of the form

\[
y_{n+r} + a_1 y_{n+r-1} + a_2 y_{n+r-2} + \ldots + a_r y_n = f(n) \quad \text{where} \quad a_1, a_2, \ldots \text{are constants.}
\]

The complete solution of a linear difference equation with constant coefficients is

\[
y_n = CF + PI
\]

Where CF is the complementary function and PI is the particular integral.

Steps for finding CF

\[
y_{n+r} + a_1 y_{n+r-1} + a_2 y_{n+r-2} + \ldots + a_r y_n = 0 \quad \text{where} \quad a_1, a_2, \ldots \text{are constants.}
\]

i. Write the equation in the symbolic form

\[
(E^r + a_1 E^{r-1} + \ldots + a_r)y_r = 0
\]

ii. Write down the auxiliary equation

\[
i.e., \quad E^r + a_1 E^{r-1} + \ldots + a_r = 0
\]

write down the solution as follows

<table>
<thead>
<tr>
<th>Roots for A.E.</th>
<th>Solution (CF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n ) (Real and distinct roots)</td>
<td>( c_1 \lambda_1^n + c_2 \lambda_2^n + \ldots + c_n \lambda_n^n ) (( c_1, c_2, \ldots, c_n ) are constants)</td>
</tr>
</tbody>
</table>
2. \( \lambda_1, \lambda_1, \lambda_3, \ldots, \lambda_n \) (2 real and equal roots) \( (c_1 + c_2n) \lambda_1^n + \lambda_3^n + \ldots \)

3. \( \lambda_1, \lambda_1, \lambda_4 \ldots \) (3 real and equal roots) \( (c_1 + c_2n + c_3n^2) \lambda_1^n + \ldots \)

4. \( \alpha + i\beta, \alpha - i\beta \ldots \)

Problems

1) Solve the difference equation \( U_{n+3} - 2U_{n+2} - 5U_{n+1} + 6U_n = 0 \)

Symbolic form of the given equation is
\[
(E^3 U_n - 2E^2 U_n - 5E U_n + 6 U_n) = 0
\]
\[
(E^3 - 2E^2 - 5E + 6) U_n = 0
\]

The auxiliary equation is
\[
E^3 - 2E^2 - 5E + 6 = 0
\]
\[
E = 1, -2, 3
\]

\[
\text{C.F.} = C_1 1^n + C_2 (-2)^n + C_3 3^n
\]

2) Solve \( U_{n+2} - 2U_{n+1} + U_n = 0 \)

Symbolic form of the given equation is
\[
(E^2 U_n - 2E U_n + U_n) = 0
\]
\[
(E^2 - 2E + 1) U_n = 0
\]

The auxiliary equation is
\[
E^2 - 2E + 1 = 0
\]
\[
E = 1, 1
\]

\[
\text{C.F.} = (C_1 + C_2 n) 1^n = C_1 + C_2 n
\]

3) Solve \( y_{n+1} - 2y_n \cos \alpha + y_{n-1} = 0 \)

Symbolic form of the given equation is
\[
(E^2 Y_{n-1} - 2E \cos \alpha Y_{n-1} + Y_{n-1}) = 0
\]
\[
(E^2 - 2E \cos \alpha + 1) Y_{n-1} = 0
\]

The auxiliary equation is
\[
E^2 - 2E \cos \alpha + 1 = 0
\]
\[
E = \cos \alpha + i \sin \alpha, \cos \alpha - i \sin \alpha
\]
\[
\text{C.F.} = y_n = r^n (C_1 \cos n\theta + C_2 \sin n\theta)
\]
\[
\theta = \tan^{-1}(\beta / \alpha) = \tan^{-1}(\sin \alpha / \cos \alpha) = \tan^{-1}(\tan \alpha) = \alpha
\]
\[
r = \sqrt{\alpha^2 + \beta^2} = \sqrt{\cos^2 \theta + \sin^2 \theta} = 1
\]
\[
\text{C.F.} = y_{n-1} = C_1 \cos (n-1)\alpha + C_2 \sin (n-1)\alpha
\]
Rules for finding the particular integral

Consider the equation,
\[ y_{n+r} + a_1 y_{n+r-1} + a_2 y_{n+r-2} + \ldots + a_r y_r = f(n) \]
The symbolic form is,
\[ \Phi(E) y_n f(n) = \ldots (1) \]
Where \( \Phi(E) = E^r + a_1 E^{r-1} + \ldots + a_r \)

Then the particular integral is given by,
\[ P.I. = \frac{1}{\Phi(E)} f(n) \]

Case 1
When \( f(n) = a^n \)
\[ P.I. = \frac{1}{\Phi(E)} a^n = \frac{1}{\Phi(E)} a^n \]
\[ \Phi(a) \quad \text{provided} \quad \Phi(a) \neq 0 \]
Replace \( E \) by \( a \)
If \( \Phi(a) = 0 \) then \( (E - a) \) must be a factor of \( \Phi(E) \) and we operate as follows,
\[ \frac{1}{\Phi(E)} a^n = na^{n-1} \]
\[ E - a \]
\[ \frac{1}{(E - a)^2} a^n = n(n-1)a^{n-2} \]
and so on.

Case 2
When \( f(n) = n^p \)
\[ P.I. = \frac{1}{\Phi(E)} n^p = \frac{1}{\Phi(E)} n^p \]
\[ = [\Phi(1 + \Delta)]^{-1} n^p \]

Case 3
If \( f(x) = \sin kn \) or \( \cos kn \)
\[ f(x) = \sin kn \]
\[ P.I. = \frac{1}{\Phi(E)} \sin kn \]
\[ = \frac{1}{\Phi(E)} \left[ e^{ikn} - e^{-ikn} \right] \]
\[ = \frac{1}{2!} \left[ \frac{1}{\Phi(E)} a^n - \frac{1}{\Phi(E)} b^n \right] \]
Where \( a = e^{ik} \) and \( b = e^{-ik} \). Now proceed as in case 1

\( f(x) = \cos kn \)
\[ P.I. = \frac{1}{\Phi(E)} \cos kn \]
\[ = \frac{1}{\Phi(E)} \left[ e^{ikn} + e^{-ikn} \right] \]
\[ = \frac{1}{2} \left[ \frac{1}{\Phi(E)} a^n - \frac{1}{\Phi(E)} b^n \right] \]
Where \( a = e^{ik} \) and \( b = e^{-ik} \). Now proceed as in case 1

Case 4
If \( f(x) = a^n f(x) \) where \( f(x) \) is a polynomial of degree \( p \) in \( x \) then
\[ P.I. = \frac{1}{\Phi(E)} a^n f(x) = \frac{a^n}{\Phi(a.E)} f(x) \]
Now proceed as in case 2
PROBLEMS

1) Solve \( y_{n+2} - 4 y_{n+1} + 3 y_n = 5^n \)
Symbolic form is \(( E^2 - 4 E +3 ) y_n = 5^n \)
The auxiliary equation is, \( E^2 - 4 E +3 = 0 \)
\[ E = 1, 3 \]
C.F. = \( C_1 1^n + C_2 3^n \)
P.I. = \( \frac{1}{5^n} = \frac{1}{5^2 - 4 \times 5 + 3} \cdot \frac{5^n}{8} \)
The complete solution is \( y_n = C_1 1^n + C_2 3^n + \frac{5^n}{8} \)

2) Solve \( y_{n+2} - 4 y_{n+1} = n^2 + n - 1 \)
Symbolic form is \(( E^2 - 4 ) y_n = n^2 + n - 1 \)
The auxiliary equation is, \( E^2 - 4 = 0 \)
\[ E = 2, -2 \]
C.F. = \( C_1 2^n + C_2 (-2)^n \)
P.I. = \( \frac{1}{E^2 - 4} \cdot \frac{1}{n^2 + n - 1} \)
\[ = \frac{1}{(1+\Delta)^2 - 4} \cdot \frac{1}{[n]^2 +2[n]-1} \]
\[ = \frac{1}{1+\Delta^2 +2\Delta-4} \cdot \frac{1}{[n]^2 +2[n]-1} \]
\[ = \frac{1}{\Delta^2 +2\Delta-3} \cdot \frac{1}{[n]^2 +2[n]-1} \]
\[ = -1 \cdot \frac{1}{3} \cdot \frac{1}{(1-\frac{\Delta^2+2\Delta}{3})^{\frac{3}{3}}} \cdot \frac{1}{[n]^2 +2[n]-1} \]
\[ = -\frac{1}{3} \cdot \frac{1}{(1-2\Delta + \frac{\Delta^2}{3})^{\frac{3}{3}}} \cdot \frac{1}{[n]^2 +2[n]-1} \]
\[ = -\frac{n^2 - 7n - 17}{3 \cdot 9 \cdot 27} \]
The complete solution is,
\[ y_n = C_1 2^n + C_2 (-2)^n - \frac{n^2 - 7n - 17}{3 \cdot 9 \cdot 27} \]
Problems

1. Solve $U_{n+2} - 4U_{n+1} + 4U_n = 2^n$
2. Solve $y_{n+2} - 4y_n = 2^n$
3. Solve $u_{n+2} - 4U_{n+1} + U_n = 3$
4. $y_{x+2} + y_x = \sin x$
5. $y_{n+2} - 4y_{n+1} - 5y_n = 3^n + 5n + 8$