Stability of Ships - Module –II (12 hours)

Transverse stability: - Form and weight stability – stability functions

Initial stability – GM0, GZ at small angles of inclinations, wall sided ships; Stability due to addition, removal and transference (horizontal, lateral and vertical) of weight, suspended weight and free surface of liquids; Stability while docking and grounding; Inclining experiment.

Large angle stability -Diagram of statical stability (GZ-curve), characteristic of GZ-curve, static equilibrium criteria; Methods for calculating the GZ-curve (Krylov, Prohaska, etc.); Cross curves of stability; Dynamical stability – diagram of dynamical stability, dynamical stability criteria

Initial Stability

- Stability when ship is upright or nearly upright (small angles)
- Indicated by the height of the metacenter (M) above (G), which is referred to as the initial metacenter height, GM
- \( GZ = GM \sin(\Phi) \)

Large Angle stability – The horizontal distance, GZ, more accurately indicates the measure of stability at angles of heel in excess of 5 degrees from the vertical. GZ is referred to as the measured of static stability in case of large angle inclinations.
- GZ not equal to GM \sin(Theta)

Moment of Statical Stability

When a ship is inclined by an external force, such as wind and wave action, the centre of buoyancy moves out to the low side, parallel to the shift of the centre of gravity of the immersed and emerged wedges, to the new centre of buoyancy of the under-water volume.

The force of buoyancy is considered to act vertically upwards through the centre of buoyancy (B), whilst the weight of the ship is considered to act vertically downwards through the centre of gravity (G).

These two equal and opposite forces produce a moment which may tend to right or capsize the ship. The moment is referred to as the moment of statical stability and may be defined as the moment to return the ship to the initial position when inclined by an external force.
The centre of buoyancy has moved from B to B1 parallel to gg1, and the force of buoyancy acts vertically upwards through B1. The weight of the ship acts vertically downwards through the centre of gravity (G).

- The perpendicular distance between the lines of action of the forces (GZ) is called the **righting lever**.
- Taking moments about the centre of gravity, the moment of statical stability is equal to the product of the righting lever and the displacement, or

\[
\text{Moment of statical stability} = W \times GZ
\]

**The moment of statical stability at a small angle of heel**

At small angles of heel the force of buoyancy may be considered to act vertically upwards through a **fixed point called the initial metacentre (M)**.

From this formula it can be seen that for any particular displacement at small angles of heel, the righting moments will vary directly as the initial metacentric height (GM).

Hence, if the ship has a comparatively large GM she will tend to be `stiff', whilst a small GM will tend to make her `tender'.

(Note - Stability of a ship depends not only upon the size of the GM or GZ but also upon the displacement). Thus two similar ships may have identical GM's, but if one is at the light displacement and the other at the load displacement, their respective states of stability will be vastly different. The ship which is at the load displacement will be much more `stiff' than the other.

**Example 1**
A ship of 4000 tonnes displacement has KG 5.5m and KM 6.0 m. Calculate the moment of statical stability when heeled 5 degrees.

\[
\begin{align*}
GM &= \text{KM} - \text{KG} = 6.0 - 5.5 = 0.5 \text{ m} \\
\text{Moment of statical stability} &= W \times GM \times \sin \theta \\
&= 4000 \times 0.5 \times \sin 5^\circ \\
\text{Moment of statical stability} &= 174.4 \text{ tonnes m}
\end{align*}
\]

**Example 2**
When a ship of 12 000 tonnes displacement is heeled 6.5 degrees the moment of statical stability is 600 tonnes m. Calculate the initial metacentric height.

\[
\begin{align*}
\text{Moment of statical stability} &= W \times GM \sin (\theta) = 600 \text{ tm} \\
\text{Moment of statical stability} &= 12 \ 000 \times GM \sin 6.5^\circ \\
\text{Therefore,} \ GM &= 0.44 \text{m}
\end{align*}
\]
The moment of statical stability at a large angle of heel

At a large angle of heel the force of buoyancy can no longer be considered to act vertically upwards through the initial metacentre (M) (see fig).

The centre of buoyancy has moved further out to the low side, and the vertical through B₁ no longer passes through (M), the initial metacentre.

- The righting lever (GZ) is once again the perpendicular distance between the vertical through G and the vertical through B₁
- Moment of statical stability is equal to \( W \times GZ \)
- GZ is no longer equal to \( GM \sin \Phi \)
- Up to the angle at which the deck edge is immersed, it may be found by using a formula known as the Wallsid ed formula. i.e.

\[
GZ = (GM + \frac{1}{2} BM \tan^2 \Phi) \sin \Phi
\]

Wall Sided Formula

A ship is said to be wall-sided if, for the angles of inclination to be considered, those portions of the outer bottom covered or uncovered by the moving waterplane are vertical with the ship upright. No practical ships are truly wall-sided, but many may be regarded as such for small angles of inclination—perhaps up to about 10 degrees.

\[
GZ = (GM + \frac{1}{2} BM \tan^2 \Phi)
\]

**Proof of wall sided formula**

Refer to the ship shown in Figure. When inclined the wedge WOW₁ is transferred to LOL₁ such that its centre of gravity shifts from g to g₁.

This causes the centre of buoyancy to shift from B to B₁.

- The horizontal components of these shifts are \( hh₁ \) and \( BB₂ \) respectively
- Vertical components being \( (gh + g₁h₁) \) and \( B₁B₂ \) respectively.

Let \( BB₂ = a \), and \( B₁B₂ = b \)

Now consider the wedge LOL₁: Area = \( \frac{1}{2} y^2 \tan \Phi \)

Consider an elementary strip longitudinally of length dx as in figure
Volume = $\frac{1}{2} y^2 \tan \Phi \ dx$

The total horizontal shift of the wedge (hh₁) = $\frac{2}{3} x \times 2 y = \frac{4}{3} y$

Moment of shifting this wedge = volume $\times$ horizontal shift = $\frac{4}{3} y \times \frac{1}{2} y^2 \tan \Phi \ dx = \frac{2}{3} y^3 \tan \Phi \ dx$

The sum of the moment of all such wedges = $\int_{0}^{L} \frac{2}{3} y^3 \tan \Phi \ dx$

But the second moment of the water-plane area about the centre-line (I) = $\int_{0}^{L} \frac{2}{3} y^3 \ dx$

∴ Sum of the moment of all such wedges = $L \tan \Phi$

$BB_2 = \frac{V \times hh_1}{V}$

or

$V \times BB_2 = V \times hh_1$

But, the sum of the moments of the wedges = $V \times hh_1$

∴ $V \times BB_2 = L \tan \Phi$

$BB_2 = \frac{1}{V} \tan \Phi$

$BB_2 = BM \tan \Phi$ - - - - - 'a'

The vertical shift of the wedge = $gh + g_1 h_1$

= $2gh$

∴ The vertical moment of the shift = $V \times 2gh$

= $2Vgh$

In figure, OL = y and Oh₁ = $\frac{2}{3} y$
But

\[ LL_1 = y \tan \theta \]

\[ \therefore g_1 h_1 = \frac{1}{3} y \tan \theta \]

The volume of the wedge = \( \frac{1}{2} y^2 \tan \theta \) dx

The moment of the vertical shift = \( \frac{1}{2} y^2 \tan \theta \) dx \times \frac{1}{3} y \tan \theta

\[ = \frac{1}{3} y^3 \tan^2 \theta \) dx\]

The vertical moment of all such wedges = \[ \int_{0}^{L} \frac{1}{3} y^3 \tan^2 \theta \) dx\]

\[ = \frac{1}{2} \tan^2 \theta \]

\[ \therefore \text{The moment of the vertical shift} = \frac{1}{3} \tan^2 \theta \]

Also

\[ B_1 B_2 = \frac{V \times 2gh}{V} \]

or

\[ V \times b = 2vgh \]

\[ \therefore V \times b = \frac{1}{2} \tan^2 \theta \]

or

\[ b = \frac{1}{V} \times \frac{\tan^2 \theta}{2} \]

\[ B_1 B_2 = \frac{BM \tan^2 \theta}{2} \quad \text{'b'} \]

Referring to Figure 14.5(a)

\[ GZ = NR \]

\[ = BR - BN \]

\[ = (BS + SR) - BN \]

\[ = a \cos \theta + b \sin \theta - BG \sin \theta \]

\[ = BM \tan \theta \cos \theta + \frac{1}{2} BM \tan^2 \theta \sin \theta - BG \sin \theta \quad \text{[from 'a' and 'b']} \]

\[ = BM \sin \theta + \frac{1}{2} BM \tan^2 \theta \sin \theta - BG \sin \theta \]

\[ = \sin \theta (BM + \frac{1}{2} BM \tan^2 \theta - BG) \]

\[ GZ = \sin \theta (GM + \frac{1}{2} BM \tan^2 \theta) \quad \text{[for } \theta \text{ up to } 25^\circ \]
Example 1

A ship of 6000 tonnes displacement has KB 3m, KM 6 m, and KG 5.5 m. Find the moment of statical stability at 25 degrees heel.

\[ GZ = (GM + \frac{1}{2} BM \tan^2 \Phi) \sin \Phi = (0.5 \times \frac{1}{2} \times 3 \times \tan^2 25) \sin 25 \]
\[ = 0.8262 \sin 25, \ GZ = 0.35m \]

Moment of statical stability = \( W \times GZ = 6000 \times 0.35 = 2100 \) tonnes m

Example 2

A box-shaped vessel 65 m \( \times \) 12 m \( \times \) 8 m has KG 4 m, and is floating in salt water upright on an even keel at 4 m draft F and A. Calculate the moments of statical stability at (a), 5 degrees and (b), 25 degrees heel.

\[ W = L \times B \times \text{draft} \times 1.025 \]
\[ = 65 \times 12 \times 4 \times 1.025 \text{ tonnes} \]
\[ W = 3198 \text{ tonnes} \]

\[ KB = \frac{1}{2} \text{ draft} = 2 \text{ m} \]
\[ BM = \frac{B^2}{12d} = \frac{12 \times 12}{12 \times 4} = 3 \text{ m} \]

At 5° heel

\[ GZ = GM \sin \theta \]
\[ = 1 \times \sin 5° \]
\[ GZ = 0.0872 \]

Moment of statical stability = \( W \times GZ \)
\[ = 3198 \times 0.0872 \]
\[ = 278.9 \text{ tonnes m} \]

At 25° heel

\[ GZ = (GM + \frac{1}{2} BM \tan^2 \theta) \sin \theta \]
\[ = (1 + \frac{1}{2} \times 3 \times \tan^2 25°) \sin 25° \]
\[ = (1 + 0.3262) \sin 25° \]
\[ = 1.3262 \sin 25° \]
\[ GZ = 0.56 \text{ metres} \]

Moment of statical stability = \( W \times GZ \)
\[ = 3198 \times 0.56 \]
\[ = 1790.9 \text{ tonnes m} \]

Ans. (a) 278.9 tonnes m and (b) 1790.9 tonnes m.
**Attwood’s formula** - The moment of statical stability at a large angle of heel may also be calculated using a formula known as Attwood’s formula: i.e.

\[
\text{Moment of statical stability} = W\left(\frac{v \times h_1}{V} - BG \sin \phi\right)
\]

**Effect of removing or discharging mass** –

The stability will be affected due to change in displacement, change in position G and B

- In each of the above figures, G represents the centre of gravity of the ship with a mass of w tonnes on board at a distance of d metres from G.
- G to G1 represents the shift of the ship’s centre of gravity due to discharging the mass w.
- In Figure (a), it will be noticed that the mass is vertically below G, and that when discharged G will move vertically upwards to G1.
- In Figure (b), the mass is vertically above G and the ship’s centre of gravity will move directly downwards to G1.
- In Figure (c), the mass is directly to starboard of G and the ship’s centre of gravity will move directly to port from G to G1.
- In Figure (d), the mass is below and to port of G, and the ship’s centre of gravity will move upwards and to starboard.

**In each case:** \( GG_1 = \frac{(w \times d)}{\text{Final displacement in metres}} \)
**Effect of adding or loading mass** – Change in displacement, G and B positions

![Diagram](image)

In each of the above figures, G represents the position of the centre of gravity of the ship before the mass of w tonnes has been loaded. After the mass has been loaded, G will move directly towards the centre of gravity of the added mass (i.e. from G to G₁).

**In each case: GG₁ = (w x d) / Final displacement in metres**

**Effect of shifting weights**

![Diagram](image)

In Figure G represents the original position of the centre of gravity of a ship with a weight of 'w' tonnes in the starboard side of the lower hold having its centre of gravity in position g₁.

If this weight is now discharged the ship's centre of gravity will move from G to G₁ directly away from g₁.

When the same weight is reloaded on deck with its centre of gravity at g₂ the ship's centre of gravity will move from G₁ to G₂.

From this it can be seen that if the weight had been shifted from g₁ to g₂ the ship's centre of gravity would have moved from G to G₂. It can also be shown that GG₂ is parallel to g₁g₂ and that

\[ GG₂ = \frac{(w \times d)}{W}, \text{metres} \]

where w is the mass of the weight shifted, d is the distance through which it is shifted, and W is the ship's displacement. The centre of gravity of the body will always move parallel to the shift of the centre of gravity of any weight moved within the body.
**Effect of suspended weights**

The centre of gravity of a body is the point through which the force of gravity may be considered to act vertically downwards. Consider the centre of gravity of a weight suspended from the head of a derrick as shown in Figure.

It can be seen that whether the ship is upright or inclined in either direction, the point in the ship through which the force of gravity may be considered to act vertically downwards is $g_1$, the point of suspension.

Thus the centre of gravity of a suspended weight is considered to be at the point of suspension.

Problem is similar to addition of weight at ‘$g_1$’.

**Conclusions**

1. The centre of gravity of a body will move directly towards the centre of gravity of any weight added.

2. The centre of gravity of a body will move directly away from the centre of gravity of any weight removed.

3. The centre of gravity of a body will move parallel to the shift of the centre of gravity of any weight moved within the body.

4. The shift of the centre of gravity of the body in each case is given by the formula: $GG_1 = \frac{w \times d}{W}$, metres

where $w$ is the mass of the weight added, removed, or shifted, $W$ is the final mass of the body, and $d$ is the distance between the centres of gravity (in 1 and 2) or the distance through which the weight is shifted (in 3).

5. When a weight is suspended its centre of gravity is considered to be at the point of suspension.
Exercise 2

1. A ship has displacement of 2400 tonnes and KG = 10.8 metres. Find the new KG if a weight of 50 tonnes mass already on board is raised 12 metres vertically.

2. A ship has displacement of 2000 tonnes and KG = 10.5 metres. Find the new KG if a weight of 40 tonnes mass already on board is shifted from the 'tween deck to the lower hold, through a distance of 4.5 metres vertically.

3. A ship of 2000 tonnes displacement has KG = 4.5 metres. A heavy lift of 20 tonnes mass is in the lower hold and has KG = 2 metres. This weight is then raised 0.5 metres clear of the tank top by a derrick whose head is 14 metres above the keel. Find the new KG of the ship.

4. A ship has a displacement of 7000 tonnes and KG = 6 metres. A heavy lift in the lower hold has KG = 3 metres and mass 40 tonnes. Find the new KG when this weight is raised through 1.5 metres vertically and is suspended by a derrick whose head is 17 metres above the keel.

5. Find the shift in the centre of gravity of a ship of 1500 tonnes displacement when a weight of 25 tonnes mass is shifted from the starboard side of the lower hold to the port side on deck through a distance of 15 metres.
Effect of free surface of liquids on stability

When a tank is completely filled with a liquid, the liquid cannot move within the tank when the ship heels. For this reason, as far as stability is concerned, the liquid may be considered a static weight having its centre of gravity at the centre of gravity of the liquid within the tank. No weights will be moved within the ship, therefore the position of G is not affected. Only the centre of buoyancy will move out to the low side.

Now consider the case where tank is only partially filled as shown in Figure. When the ship heels, the liquid flows to the low side of the tank such that its centre of gravity shifts from g to \( g_1 \). This will cause the ship's centre of gravity to shift from G to \( G_1 \), parallel to \( gg_1 \).

\[
\text{Moment of statical stability} = W \times G_1 Z_1 = W \times GvZv = W \times GvM \sin \Phi
\]

This indicates that the effect of the free surface is to reduce the effective metacentric height from \( GM \) to \( GvM \). \( GvM \) is therefore the virtual loss of \( GM \) due to the free surface. Any loss in \( GM \) is a loss in stability.

This loss takes place irrespective of the position of the free surface in the ship. Any loss in \( GM \) decreases the ship’s stability. It should also be noted from Figure that even though the distance \( GG_1 \) is fairly small it produces a relatively large virtual loss in \( GM \) (\( GvM \)).

If free surface be created in a ship with a small initial metacentric height, the virtual loss of \( GM \) due to the free surface may result in a negative metacentric height. This would cause the ship to take up an angle of loll which may be dangerous and in any case is undesirable. This should be borne in mind when considering whether or not to run water ballast into tanks to correct an angle of loll, or to increase the \( GM \). Until the tank is full there will be a virtual loss of \( GM \) due to the free surface effect of the liquid.

Calculating the effect of free surface of liquids (FSE)

When a tank is partially filled with a liquid, the ship suffers a virtual loss in metacentric height which can be calculated by using the formula

\[
FSE = \text{Virtual loss of } GM = \frac{i}{W} \times \rho \times \frac{n}{n^2} \text{ metres}
\]

where
- \( i \) = the second moment of the free surface about the centre line, in m^4
- \( W \) = the ship’s displacement, in tonnes
- \( \rho \) = the density of the liquid in the tank, in tonnes/cu.m
- \( n \) = the number of the longitudinal compartments into which the tank is subdivided
- \( i \times \rho \) = free surface moment, in tonnes m.
When the ship is inclined, a wedge of the liquid in the tank will shift from the high side to the low side such that its centre of gravity shifts from \( G \) to \( G_1 \). This will cause the centre of gravity of the ship to shift from \( G \) to \( G_1 \), where

\[
GG_1 = \frac{W \times \rho_1}{W} \times g_1
\]

Let

\[
\rho_1 = \text{density of the liquid in the tank}
\]

and

\[
\rho_2 = \text{density of the water in which the ship floats}
\]

then

\[
GG_1 = \frac{V \times \rho_1 \times g_1}{V \times \rho_2}
\]

Had there been no free surface when the ship inclined, the righting lever would have been \( GZ \). But, due to the liquid shifting, the righting lever is reduced to \( G_1Z_1 \) or \( G_1Z_v \). The virtual reduction of \( GM \) is therefore \( GG_v \).

For a small angle of heel:

\[
GG_1 = GG_v \times \theta
\]

\[
\therefore \quad GG_v \times \theta = \frac{V \times g_1 \times \rho_1}{V \times \rho_2}
\]

or

\[
GG_v = \frac{V \times g_1 \times \rho_1}{V \times \theta \times \rho_2}
\]

From the proof of \( BM = 1/V, I \times \theta = V \times g_1 \)

Let \( i \) = the second moment of the free surface about the centre line.

Then

\[
GG_v = \frac{i \times \rho_1}{V \times \rho_2}
\]

This is the formula to find the virtual loss of \( GM \) due to the free surface effect in an undivided tank.

Now assume that the tank is subdivided longitudinally into \( n \) compartments of equal width as shown in Figure 20.1(b).
Let

\[
l = \text{length of the tank, and}
\]

\[
b = \text{breadth of the tank.}
\]

The breadth of the free surface in each compartment is thus \( b/n \), and the second moment of each free surface is given by

\[
\frac{l \times (b/n)^3}{12}
\]
From this formula it can be seen that, when a tank is subdivided longitudinally, the virtual loss of GM for the undivided tank is divided by the square of the number of compartments into which the tank is divided. Also note that the actual weight of the liquid in the tank will have no effect whatsoever on the virtual loss of GM due to the free surface.
Note. Transverse subdivisions in partially filled tanks (slack tanks) do not have any significant influence on reducing free surface effects.

However, fitting longitudinal bulkheads do have a very effective influence in reducing this virtual loss in GM

**Example**
A ship of 8153.75 tonnes displacement has KM = 8m, KG = 7.5 m, and has a double bottom tank 15mx10mx2m which is full of salt water ballast. Find the new GM if this tank is now pumped out till half empty.

Note. The mass of the water pumped out will cause an actual rise in the position of the ship's centre of gravity and the free surface created will cause a virtual loss in GM. There are therefore two shifts in the position of the centre of gravity to consider.

Let $GG_1$ represent the actual rise of G due to the mass discharged.

The mass of water discharged ($w$) = $15 \times 10 \times 1 \times 1.025$ tonnes

$$w = 153.75 \text{ tonnes}$$

$$W_2 = W_1 - w = 8153.75 - 153.75$$

$$= 8000 \text{ tonnes}$$

$$GG_1 = \frac{w \times d}{W_2}$$

$$= \frac{153.75 \times 6}{8000}$$

$$GG_1 = 0.115 \text{ m}$$
Let $G_1G_v$ represent the virtual loss of GM due to free surface or rise in $G_1$.

Then:

$$G_1G_v = \frac{i}{W} \times \rho_1 \times \frac{1}{n^2}$$

as per equation (I)

$$n = 1$$

$$\therefore G_1G_v = \frac{i}{W_2} \times \rho_{SW}$$

or

$$G_1G_v = \frac{1b^3}{12} \times \frac{\rho_{SW}}{W_2} \times \frac{1}{n^3}$$

Loss in GM = 0.160 m

or

$$G_1G_v = \frac{15 \times 10^3 \times 1.025}{12 \times 8000} = 0.160 \text{ m}$$

Old KM = 8.000 m

Old KG = 7.500 m

Old GM = 0.500 m

Actual rise of $G = 0.115$ m

0.385 m = $GM_{solid}$

$G_1G_v = \text{Virtual rise of } G = 0.160 \text{ m}$

$$= 0.225 \text{ m}$$

**Ans.** New GM = 0.225 m = $GM_{fluid}$

---

**Example 2**

A ship of 6000 tonnes displacement, floating in salt water, has a double bottom tank $20 \text{ m} \times 12 \text{ m} \times 2 \text{ m}$ which is divided at the centre line and is partially filled with oil of relative density 0.82. Find the virtual loss of GM due to the free surface of the oil.

$$\text{Virtual loss of GM} = \frac{i}{W} \times \rho_{oil} \times \frac{1}{n^2}$$

$$= \frac{1b^3}{12} \times \rho_{oil} \times \frac{1}{W} \times \frac{1}{n^2}$$

$$= \frac{20 \times 12^3}{12 \times 6000} \times 0.820 \times \frac{1}{2^2}$$

**Ans.** Virtual loss of GM = 0.098 metres
Question: A ship has a displacement of 3000 tonnes. On the vessel is a rectangular double-bottom tank 15 m long and 8 m wide. This tank is partially filled with ballast water having a density of 1.025 t/m³.
If the \( GM_T \) without free surface effects is 0.18 m calculate the virtual loss in \( GM_T \) and the final \( GM_T \) when the double bottom tank has:

(a) no divisional bulkheads fitted;
(b) one transverse bulkhead fitted at mid-length;
(c) one longitudinal bulkhead fitted on 4. of the tank;
(d) two longitudinal bulkheads fitted giving three equal divisions.

Answer

\[ l = 15 \text{ m} \]
\[ b = 8 \text{ m} \]

![Fig. 20.4(a)](image)

FSE = virtual loss in \( GM_T \) or rise in \( G = \frac{l \times \rho_{SW}}{W} \)

\[ = \frac{l \times b \times \rho_{SW}}{12 \times W} \] (see Fig. 20.4(a))

.: virtual loss in \( GM_T \) = \( \frac{15 \times 8 \times 1.025}{3000 \times 12} \)

\[ = 0.02187 \text{ m} \uparrow \]

.: \( GM_T \) finally = 0.1800 − 0.2187

\[ = -0.0387 \text{ m} \uparrow \]

i.e. unstable ship!!
FSE = virtual loss in $GM_T$ or rise in $G = \frac{2 \cdot l_2 \times b_2^3}{12 \times W} \times \rho_{SW}$ (see Fig. 20.4(b))

\[ \therefore \text{virtual loss} = \frac{2 \times 7.5 \times 8^3 \times 1.025}{12 \times 3000} \]

\[ = 0.2187 \text{ m} \uparrow \]

This is same answer as for part (a). Consequently it can be concluded that fitting transverse divisional bulkheads in tanks does not reduce the free surface effects. Ship is still unstable!!

FSE = vertical loss in $GM_T$ or rise in $G = \frac{2 \cdot l_1 b_2^3}{12 \times W} \rho_{SW}$

\[ \therefore \text{virtual loss in } GM_T = \frac{2 \times 15 \times 4^3 \times 1.025}{12 \times 3000} \quad \text{(see Fig. 20.4(c))} \]

\[ = 0.0547 \text{ m} \uparrow \quad \text{i.e. } \frac{1}{4} \text{ of answer to part (a)} \]

Hence

final $GM_T = 0.1800 - 0.0547 = +0.1253 \text{ m}$ Ship is stable.

\[ l_1 = 15 \text{ m} \]
\[ b_3 = 8/3 \text{ m} \]
\[ b_2 = 8/3 \text{ m} \]
\[ b_1 = 8/3 \text{ m} \]

Fig. 20.4(d)

$GM_T$ is now $+$ve, but below the minimum $GM_T$ of 0.15 m that is allowable by D.Tp. regulations.

FSE = virtual loss in $GM_T$ or rise in $G = \frac{3 \cdot l_1 \times b_3^3}{12 \times W} \times \rho_{SW}$ (see Fig. 20.4(d))

\[ \therefore \text{Virtual loss in } GM_T = \frac{3 \times 15 \times \left(\frac{8}{3}\right)^3 \times 1.025}{12 \times W} \]

\[ = 0.0243 \text{ m} \uparrow \quad \text{i.e. } \frac{1}{9} \text{ of answer to part (a)} \]

Hence

final $GM_T = 0.1800 - 0.0243 = +0.1557 \text{ m}$ ship is stable.
So longitudinal divisional bulkheads (watertight or wash-bulkheads) are effective. They cut down rapidly the loss in GMT. Note the 1/n² law where n is the number of equal divisions made by the longitudinal bulkheads.

Free surface effects therefore depend on:

(I) density of slack liquid in the tank;
(II) ship's displacement in tonnes;
(III) dimensions and shape of the slack tanks;
(IV) bulkhead subdivision within the slack tanks.

The longitudinal divisional bulkheads referred to in examples in this chapter need not be absolutely watertight; they could have openings in them. Examples on board ship are the centreline wash bulkhead in the Fore Peak tank and in Aft Peak tank.
**Inclining Experiment**

The initial conditions must be known to determine the stability of a ship in any particular condition of loading.

This means knowing the ship's lightweight, the VCG or KG at this lightweight, plus the LCG for this lightweight measured from amidships are to be known.

**It is in order to find the KG for the light condition the Inclining Experiment is performed.**

Lightship condition is a ship complete in all respects, but without consumables, stores, cargo, crew and effects, and without any liquids on board except that machinery and piping fluids, such as lubricants and hydraulics, are at operating levels.

The experiment is carried out by the builders when the ship is as near to completion as possible; that is, as near to the light condition as possible. If a ship has undergone major repair or refit, she should then have an Inclining Experiment to obtain her modified Lightweight and centre of gravity (VCG and LCG).

The ship is forcibly inclined by shifting weights a fixed distance across the deck. The weights used are usually concrete blocks, and the inclination is measured by the movement of plumb lines across specially constructed battens which lie perfectly horizontal when the ship is upright. Usually two or three plumb lines are used and each is attached at the centre line of the ship at a height of about 10m above the batten. If two lines are used then one is placed forward and the other aft. If a third line is used it is usually placed amidships. For simplicity, in the following explanation only one weight and one plumb line is considered.

**What are the conditions or pre-requisites for conduct of inclining experiment**

(necessary to ensure that the KG obtained is as accurate as possible)

- There should be little or no wind, as this may influence the inclination of the ship. If there is any wind the ship should be head on or stern on to it.
- The ship should be floating freely. This means that nothing outside the ship should prevent her from listing freely. There should be no barges or lighters alongside; mooring ropes should be slacked right down, and there should be plenty of water under the ship to ensure that at no time during the experiment will she touch the bottom.
- Any loose weights within the ship should be removed or secured in place.
- There must be no free surfaces within the ship. Bilges should be dry. Boilers and tanks should be completely full or empty.
- Any persons not directly concerned with the experiment should be sent ashore.
- The ship must be upright at the commencement of the experiment.
- A note of `weights on' and `weights off' to complete the ship each with a VCG and LCG (from midships).
**Procedure**

A weight is shifted across the deck transversely, causing the ship to list. A little time is allowed for the ship to settle and then the deflection of the plumb line along the batten is noted. If the weight is now returned to its original position the ship will return to the upright. She may now be listed in the opposite direction. From the deflections the GM is obtained as follows. In

- let a mass of `w' tonnes be shifted across the deck through a distance `d' metres.
- This will cause the centre of gravity of the ship to move from G to G₁ parallel to the shift of the centre of gravity of the weight.
- The ship will then list to bring G₁ vertically under M, i.e. to θ degrees list. The plumb line will thus be deflected along the batten from B to C.
- Since AC is the new vertical, angle BAC must also be θ degrees.
- We can write, \( \cot \theta = \frac{GM}{GG_1} = \frac{AB}{BC} \)

or \( GM = GG_1 \times \frac{AB}{BC} \)

\[
GG_1 = \frac{w \times d}{W}
\]

\[
GM = \frac{w \times d}{W} \times \frac{AB}{BC}
\]

\[
GM = \frac{w \times d}{W \tan \theta}
\]

\[
KG = KM - GM
\]

In this formula,
- AB = the length of the plumb line and BC = the deflection along the batten can be measured.
- `w' = the mass shifted, `d' = the distance through which it was shifted, and `W' = ship's displacement, will all be known.
- Hydrostatics - W and KM for this draft is known.
- The GM can therefore be calculated using the formula. and hence the present KG is found
- By taking moments about the keel, allowance can now be made for weights which must be loaded or discharged to bring the ship to the light condition. In this way the light KG is found.
Example 1
When a mass of 25 tonnes is shifted 15 m transversely across the deck of a ship of 8000 tonnes displacement, it causes a deflection of 20 cm in a plumb line 4 m long. If the KM = 7 m, calculate the KG.

Fig. 26.2

\[
\frac{GM}{GG_1} = \frac{AB}{BC} = \frac{1}{\tan \theta}
\]

\[
\therefore \tan \theta \ GM = GG_1
\]

\[
GM = \frac{w \times d}{W} \times \frac{1}{\tan \theta}
\]

\[
= \frac{25 \times 15}{8000} \times \frac{4}{0.2}
\]

\[
GM = 0.94 \text{ m}
\]

KM = 7.00 m

**Ans.** KG = 6.06 m as shown in sketch on page 240.
Example 2
When a mass of 10 tonnes is shifted 12 m transversely across the deck of a ship with a GM of 0.6 m it causes 0.25 m deflection in a 10 m plumb line. Calculate the ship’s displacement.

![Diagram](Image)

\[ GM = \frac{w \times d}{W} \times \frac{1}{\tan \theta} \]
\[ W = \frac{w \times d}{GM} \times \frac{1}{\tan \theta} \]
\[ = \frac{10 \times 12 \times 10}{0.6 \times 0.25} \]
\[ W = 8000. \]

*Ans.* Displacement = 8000 tonnes

Example 3
When a vessel of 5300 tonnes displacement KM 7.7 m is inclined by shifting 10 tonnes 16 m, it is noted that the mean deflection of a plumb line 12 m long is 33.25 cm. What is her KG and inclined angle \( \theta \)?

\[ GM = \frac{w \times d}{W \tan \theta} \]
\[ \therefore GM = \frac{10 \times 16 \times 12 \times 100}{5300 \times 33.25} \]
\[ \therefore GM = 1.09 \text{ m} \]

KM = 7.7 m

KG = 6.61 m

\[ \tan \theta = \frac{x}{l} = \frac{33.25}{1200} = 0.0277 \quad \therefore \theta = 1.59^\circ \]
**Example 4.** A ship of 6000 tonnef displacement is inclined. The ballast weights are arranged in four equal units of 10 tonnef and each is moved transversely through a distance of 10 m. The pendulum deflections recorded are 31, 63, 1, −30, −62 and 0 cm with a pendulum length of 7 m. Calculate the metacentric height at the time of the experiment.

**Solution:** Since the moment is applied in equal increments of 100 tonnef m, each such increment can be represented by unity in the table as below, using the first method of analysis:

<table>
<thead>
<tr>
<th>Applied moment</th>
<th>$m^2$</th>
<th>Deflection</th>
<th>$m \times d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units of 100 tonnef m</td>
<td></td>
<td>d cm</td>
<td></td>
</tr>
<tr>
<td>$m$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>31</td>
<td>31</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>63</td>
<td>126</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>−1</td>
<td>1</td>
<td>−30</td>
<td>30</td>
</tr>
<tr>
<td>−2</td>
<td>4</td>
<td>−62</td>
<td>124</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \sum m^2 = 10 \quad \sum md = 311 \]

:. Mean deflection per unit moment = 31.1 cm, i.e. the pendulum is deflected through 31.1 cm for each 100 tonnef m of applied moment

\[
\text{Angle of heel} = \frac{\text{Pendulum deflection}}{\text{Pendulum length}} = \frac{31.1}{700} = 0.0444 \text{ radians.}
\]

For small angles

\[ \text{Applied moment} = \Delta \text{GM} \phi \]

:. \[ 100 = 6000 \overline{\text{GM}} \times 0.0444 \]

\[ \therefore \overline{\text{GM}} = 0.375 \text{ m} \]

Then KG = KM - GM
**Statistical Stability curves (GZ Curve)**

The curve of statistical stability for a ship in any particular condition of loading is obtained by plotting the righting levers against angle of heel as shown in Figure.

![GZ Curve Diagram]

**Stability information from GZ Curve**

**The range of stability.** This is the range over which the ship has positive righting levers. In Figure range is from 0 degrees to 86 degrees.

**The angle of vanishing stability.** This is the angle of heel at which the righting lever returns to zero, or is the angle of heel at which the sign of the righting levers changes from positive to negative. The angle of vanishing stability in the figure is 86 degrees.

**The maximum GZ** is obtained by drawing a tangent to the highest point in the curve. In Figure AB is the tangent and this indicates a maximum GZ of 0.63 metres. If a perpendicular is dropped from the point of tangency, it cuts the heel scale at the angle of heel at which the maximum GZ occurs. In the present case the maximum GZ occurs at 42 degrees heel.

**The initial metacentric height (GM)** is found by drawing a tangent to the curve through the origin (OX in Figure), and then erecting a perpendicular through an angle of heel of 57.3 degrees (1 radian). Let the two lines intersect at Y. Then the height of the intersection above the base (YZ), when measured on the GZ scale, will give the initial metacentric height. In the present example the GM is 0.54 metres.

**Angle of Loll**

A ship having a negative initial metacentric height will be unstable when inclined to a small angle. As the angle of heel increases, the centre of buoyancy will move out still further to the low side. If the centre of buoyancy moves out to a position vertically under G, the capsizing moment may disappear. **The angle of heel at which this occurs is called the angle of loll.** It will be noticed that at the angle of loll, the GZ is zero. G remains on the centre line.

![Angle of Loll Diagram]

Figure shows the stability curve for a ship having a negative initial metacentric height.

At angles of heel of less than 18 degrees the righting levers are negative, whilst at angles of heel between 18 degrees and 90 degrees the levers are positive.
• The angle of loll in this case is 18 degrees
• the range of stability is 18 degrees to 90 degrees, and the angle of vanishing stability is 90 degrees.
• -ve GM is plotted at 57.3°.

From this it can be seen that the ship will oscillate about the angle of loll instead of about the vertical. If the centre of buoyancy does not move out far enough to get vertically under G, the ship will capsize. The angle of loll will be to port or starboard and back to port depending on external forces such as wind and waves. One minute it may hop over to port and then suddenly flop over to Starboard.

There is always the danger that G will rise above M and create a situation of unstable equilibrium. This will cause capsizing of the ship.

**Correcting an angle of loll**

If a ship takes up an angle of loll due to a very small negative GM it should be corrected as soon as possible. (GM may be, for example -0.05 to -0.10 m, well below the minimum stipulated criteria).

• First make sure that the heel is due to a negative GM and not due to uneven distribution of the weights on board.
• Lower the position of G (effective centre of gravity) sufficiently to bring it below the initial metacentre.
• Any slack tanks should be topped up to eliminate the virtual rise of G due to free surface effect.
• If there are any weights which can be lowered within the ship, they should be lowered. (For example, derricks may be lowered if any are topped; oil in deep tanks may be transferred to double bottom tanks, etc.).
• Fill lower tanks one at a time to reduce free surface effects.
Cross Curves of Stability (KN Curves)

These are a set of curves from which the righting lever about an assumed centre of gravity (usually at K) for any angle of heel at any particular displacement may be found.

KN - the righting lever measured from the keel.

\[ GZ = XN = KN - KX \]

or

\[ GZ = KN - KG \sin \theta \]  

Thus, the righting lever is found by always subtracting from the KN ordinate a correction equal to KG \( \sin \theta \).

X axis – Displacements
Y axis – KN in meters

To obtain the righting levers (GZs) for a particular displacement and KG, locate the displacement concerned on the x axis. Through this point erect a perpendicular to cut all the curves and values of KN are obtained. The correct righting levers are then obtained by subtracting from the KN values a correction equal to the product of the KG and \( \sin \theta \).
Example

Find the righting levers for M.V. ‘Cargo-Carrier’ when the displacement is 40,000 tonnes and the KG is 10 metres.

<table>
<thead>
<tr>
<th>Heel (°)</th>
<th>KN</th>
<th>sin θ</th>
<th>KG sin θ</th>
<th>GZ = KN - KG sin θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>5°</td>
<td>0.90</td>
<td>0.087</td>
<td>0.87</td>
<td>0.03</td>
</tr>
<tr>
<td>10°</td>
<td>1.92</td>
<td>0.174</td>
<td>1.74</td>
<td>0.18</td>
</tr>
<tr>
<td>15°</td>
<td>3.11</td>
<td>0.259</td>
<td>2.59</td>
<td>0.52</td>
</tr>
<tr>
<td>20°</td>
<td>4.25</td>
<td>0.342</td>
<td>3.42</td>
<td>0.83</td>
</tr>
<tr>
<td>30°</td>
<td>6.30</td>
<td>0.500</td>
<td>5.00</td>
<td>1.30</td>
</tr>
<tr>
<td>45°</td>
<td>8.44</td>
<td>0.707</td>
<td>7.07</td>
<td>1.37</td>
</tr>
<tr>
<td>60°</td>
<td>9.39</td>
<td>0.866</td>
<td>8.66</td>
<td>0.73</td>
</tr>
<tr>
<td>75°</td>
<td>9.29</td>
<td>0.966</td>
<td>9.66</td>
<td>-0.37</td>
</tr>
<tr>
<td>90°</td>
<td>8.50</td>
<td>1.000</td>
<td>10.00</td>
<td>-1.50</td>
</tr>
</tbody>
</table>

Dynamical Stability

Dynamical stability is defined as the work done in inclining a ship.

The dynamical stability to any angle of heel may be found by multiplying the area under the statical stability curve (GZ curve) to the angle concerned by the vessel’s displacement. i.e.

Dynamical stability = W x Area under the stability curve

\[ \therefore \text{Dynamical stability} = \int_{0}^{\theta} W \times GZ \times d\theta \]

\[ = W \int_{0}^{\theta} GZ \, d\theta \]

Therefore the dynamical stability to any angle of heel is found by multiplying the area under the statical stability curve to that angle by the displacement.
Consider the ship shown in Figure. When the ship is upright the force \( W \) acts upwards through B and downwards through G.

\[
\text{Work done} = \text{Weight} \times \text{Vertical separation of G and B}
\]

or

\[
\text{Dynamical stability} = W \left( B;Z - BG \right)
\]

\[
\text{or} \quad W \left( B;R + RZ - BG \right)
\]

Dynamical stability = \( W \left[ \frac{v(gh + g_1h_1)}{V} - BG(1 - \cos \theta) \right] \)

This is known as Moseley’s formula for dynamical stability.

**Example 1**

A ship of 5000 tonnes displacement has righting levers as follows:

<table>
<thead>
<tr>
<th>Angle of heel</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
</tr>
</thead>
<tbody>
<tr>
<td>GZ (metres)</td>
<td>0.21</td>
<td>0.33</td>
<td>0.40</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Calculate the dynamical stability to 40 degrees heel.

<table>
<thead>
<tr>
<th>GZ</th>
<th>SM</th>
<th>Functions of area</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.21</td>
<td>4</td>
<td>0.84</td>
</tr>
<tr>
<td>0.33</td>
<td>2</td>
<td>0.66</td>
</tr>
<tr>
<td>0.40</td>
<td>4</td>
<td>1.60</td>
</tr>
<tr>
<td>0.43</td>
<td>1</td>
<td>0.43</td>
</tr>
</tbody>
</table>

\[ 3.53 = \Sigma_l \]

\[ h = 10^\circ \]

\[ h = \frac{10}{57.3} \text{ radians} = \text{common interval CI} \]

The area under the stability curve = \( \frac{1}{3} \times \text{CI} \times \Sigma_l \)

\[ = \frac{1}{3} \times \frac{10}{57.3} \times 3.53 \]

\[ = 0.2053 \text{ metre-radians} \]

Dynamical stability = \( W \times \text{Area under the stability curve} \)

\[ = 5000 \times 0.2053 \]

**Ans.** Dynamical stability = 1026.5 metre tonnes
\[ h = 10^\circ \]

\[ h = \frac{10}{57.3} \text{ radians = common interval CI} \]

The area under the stability curve:

\[ \frac{1}{3} \times \text{Cl} \times \Sigma_1 \]

\[ = \frac{1}{3} \times \frac{10}{57.3} \times 3.53 \]

\[ = 0.2053 \text{ metre-radians} \]

Dynamical stability = \( W \times \text{Area under the stability curve} \)

\[ = 5000 \times 0.2053 \]

\[ \text{Ans. Dynamical stability} = 1026.5 \text{ metre tonnes} \]

At 10° heel:

\[ \text{GZ = GM} \times \sin \theta \]

\[ = 0.6 \times \sin 10^\circ \]

\[ \text{GZ} = 0.104 \text{ m} \]

At 20° heel:

\[ \text{GZ} = (\text{GM} + \frac{1}{2} \text{BM} \tan^2 \theta) \sin \theta \]

\[ = (0.6 + \frac{1}{2} \times 2.08 \times \tan^2 20^\circ) \sin 20^\circ \]

\[ = (0.6 + 0.138) \sin 20^\circ \]

\[ = 0.738 \sin 20^\circ \]

\[ \text{GZ} = 0.252 \text{ m} \]

Area under the curve:

\[ \frac{1}{3} \times \text{Cl} \times \Sigma_1 \]

\[ \frac{1}{3} \times \frac{10}{57.3} \times 0.668 \]

Area under the curve = 0.0389 metre radians

Dynamical stability = \( W \times \text{Area under the curve} \)

\[ = 1845 \times 0.0389 \]

\[ \text{Ans. Dynamical stability} = 71.77 \text{ m tonnes} \]
**Dry docking and grounding**

When a ship enters a drydock she must have a positive initial GM, be upright, and trimmed slightly, usually by the stern. The dock gates are then closed and pumping out commences.

As the water level falls in the drydock there is no effect on the ship's stability so long as the ship is completely waterborne, but after the stern lands on the blocks the draft aft will decrease and the trim will change by the head.

This will continue until the ship takes the blocks overall throughout her length, when the draft will then decrease uniformly forward and aft.

The interval of time between the stern post landing on the blocks and the ship taking the blocks overall is referred to as the critical period. During this period part of the weight of the ship is being borne by the blocks, and this creates an upthrust at the stern which increases as the water level falls in the drydock.

The upthrust causes a virtual loss in metacentric height and it is essential that positive effective metacentric height be maintained throughout the critical period, or the ship will heel over and perhaps slip off the blocks with disastrous results.

![Diagram showing longitudinal section of a ship during the critical period](image)

Fig. 28.1

Therefore,

\[ P \times l = \text{MCTC} \times t \]

or,

\[ P = \frac{\text{MCTC} \times t}{l} \]

where

- \( P \) = the upthrust at the stern in tonnes,
- \( t \) = the change of trim since entering the drydock in centimetres, and
- \( l \) = the distance of the centre of flotation from aft in metres.

Figure shows the longitudinal section of a ship during the critical period. 'P' is the upthrust at the stern and 'l' is the distance of the centre of flotation from aft. The trimming moment is given by PxI.

But the trimming moment is also equal to MCTC x Change of trim.